PHYSICS
AND
FRACTAL STRUCTURES

J.-F. GOUYET

Laboratoire de Physique de la Matière Condensée
Ecole Polytechnique

Foreword

When intellectual and political movements ponder their roots, no event looms larger than the first congress. The first meeting on fractals was held in July 1982 in Courchevel, in the French Alps, through the initiative of Herbert Budd and with the support of IBM Europe Institute. Jean-François Gouyet's book reminds me of Courchevel, because it was there that I made the acquaintance and sealed the friendship of one of the participants, Bernard Sapoval, and it was from there that the fractal bug was taken to Ecole Polytechnique. Sapoval, Gouyet and Michel Rosso soon undertook the work that made their laboratory an internationally recognized center for fractal research. If I am recounting all this, it is to underline that Gouyet is not merely the author of a new textbook, but an active player on a world-famous stage. While the tone is straightforward, as befits a textbook, he speaks with authority and deserves to be heard.

The topic of fractal diffusion fronts which brought great renown to Gouyet and his colleagues at Polytechnique is hard to classify, so numerous and varied are the fields to which it applies. I find this feature to be particularly attractive. The discovery of fractal diffusion fronts can indeed be said to concern the theory of welding, where it found its original motivation. But it can also be said to concern the physics of (poorly) condensed matter. Finally it also concerns one of the most fundamental concepts of mathematics, namely, diffusion. Ever since the time of Fourier and then of Bachelier (1900) and Wiener (1922), the study of diffusion keeps moving forward, yet entirely new questions come about rarely. Diffusion fronts brought in something entirely new.

Returning to the book itself, if the variety of the topics comes as a surprise to the reader, and if the brevity of some of treatments leaves him or her hungry for more, then the author will have achieved the goal he set himself. The most important specialized texts treating the subject are carefully referenced and should satisfy most needs.

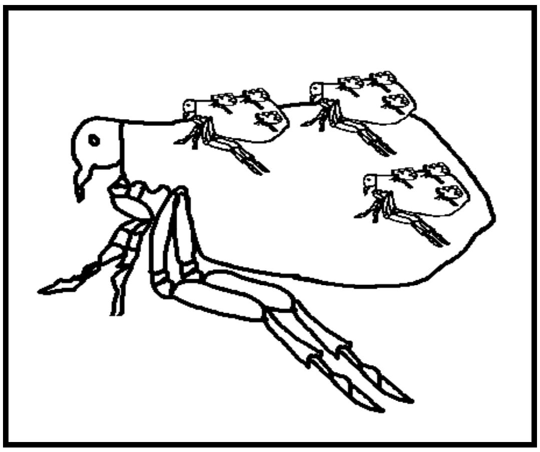
To sum up, I congratulate Jean-François warmly and wish his book the great success it deserves.

Benoît B. MANDELBROT

Yale University
IBM T.J. Watson Research Center

…
So, Nat'ralists observe, a Flea
Hath smaller Fleas that on him prey,
And these have smaller yet to bite 'em
And so proceed ad infinitum.
…

Jonathan Swift, 1733,
On poetry, a Rhapsody.



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Preface

The introduction of the concept of fractals by Benoît B. Mandelbrot at the beginning of the 1970's represented a major revolution in various areas of physics. The problems posed by phenomena involving fractal structures may be very difficult, but the formulation and geometric understanding of these objects has been simplified considerably. This no doubt explains the immense success of this concept in dealing with all phenomena in which a semblance of disorder appears.

Fractal structures were discovered by mathematicians over a century ago and have been used as subtle examples of continuous but *nonrectifiable* curves, that is, those whose length cannot be measured, or of continuous but *nowhere differentiable* curves, that is, those for which it is impossible to draw a tangent at any their points. Benoît Mandelbrot was the first to realize that many shapes in nature exhibit a fractal structure, from clouds, trees, mountains, certain plants, rivers and coastlines to the distribution of the craters on the moon. The existence of such structures in nature stems from the presence of disorder, or results from a functional optimization. Indeed, this is how trees and lungs maximise their surface/volume ratios.

This volume, which derives from a course given for the last three years at the Ecole Supérieure d'Electricité, should be seen as an introduction to the numerous phenomena giving rise to fractal structures. It is intended for students and for all those wishing to initiate themselves into this fascinating field where apparently disordered forms become geometry. It should also be useful to researchers, physicists, and chemists, who are not yet experts in this field.

This book does not claim to be an exhaustive study of all the latest research in the field, yet it does contains all the material necessary to allow the reader to tackle it. Deeper studies may be found not only in Mandelbrot's books (Springer Verlag will publish a selection of books which bring together reprints of published articles along with many unpublished papers), but also in the very abundant, specialized existing literature, the principal references of which are located at the end of this book.

The initial chapter introduces the principal mathematical concepts needed to characterize fractal structures. The next two chapters are given over to fractal geometries found in nature; the division of these two chapters is intended to help the presentation. Chapter 2 concerns those structures which may extend to enormous sizes (galaxies, mountainous reliefs, etc.), while Chap. 3 explains those fractal structures studied by materials physicists. This classification is obviously too rigid; for example, fractures generate similar structures ranging in size from several microns to several hundreds of meters.

In these two chapters devoted to fractal geometries produced by the physical world, we have introduced some very general models. Thus fractional Brownian motion is introduced to deal with reliefs, and percolation to deal with disordered media. This approach, which may seem slightly unorthodox seeing that these concepts have a much wider range of application than the examples to which they are attached, is intended to lighten the mathematical part of the subject by integrating it into a physical context.

Chapter 4 concerns growth models. These display too great a diversity and richness to be dispersed in the course of the treatment of the various phenomena described.

Finally, Chap. 5 introduces the dynamic aspects of transport in fractal media. Thus it completes the geometric aspects of dynamic phenomena described in the previous chapters.

I would like to thank my colleagues Pierre Collet, Eric Courtens, François Devreux, Marie Farge, Max Kolb, Roland Lenormand, Jean-Marc Luck, Laurent Malier, Jacques Peyrière, Bernard Sapoval, and Richard Schaeffer, for the many discussions which we have had during the writing of this book. I thank Benoît Mandelbrot for the many improvements he has suggested throughout this book and for agreeing to write the preface. I am especially grateful to Etienne Guyon, Jean-Pierre Hulin, Pierre Moussa, and Michel Rosso for all the remarks and suggestions that they have made to me and for the time they have spent in checking my manuscript. Finally, I would like to thank Marc Donnart and Suzanne Gouyet for their invaluable assistance during the preparation of the final version.

The success of the French original version published by Masson, has motivated Masson and Springer to publish the present English translation. I am greatly indebted to them. I acknowledge Dr. David Corfield who carried out this translation and Dr. Clarissa Javanaud and Prof. Eugene Stanley for many valuable remarks upon the final translation. During the last four years, the use of fractals has widely spread in various fields of science and technology, and some new approaches (such as wavelets transform) or concepts (such as scale relativity) have appeared. But the essential of fractal knowledge was already present at the end of the 1980s.

Palaiseau, July 1995